Quiz 2 review key

Stat 301

Summer 2019

(1) Real estate ads suggest that 64% of homes for sale have garages, 21% have swimming pools, and 17% have both. Find the following probabilities:

(a) No garage:
$P(garage') = 1 - P(garage) = 1 - .64 = 0.36$

(b) No pool:
$P(pool') = 1 - P(pool) = 1 - .21 = 0.79$

(c) Have a pool given there is a garage:
$P(pool|garage) = \frac{P(garage \cap pool)}{P(garage)} = \frac{0.17}{0.64} = 0.265625$

(d) Pool or garage:
$P(pool \cup garage) = P(pool) + P(garage) - P(pool \cap garage) = 0.21 + 0.64 - 0.17 = 0.68$

(e) Neither a pool nor a garage (make matrix and value is in matrix):
$P(garage' \cap pool') = 0.32$ OR $= 1 - P(pool \cup garage) = 1 - 0.68 = 0.32$

(f) Have a pool given there is no garage:
$P(pool|garage') = \frac{P(garage' \cap pool)}{P(garage')} = \frac{0.04}{0.21} = 0.1904762$

(g) Are having a pool and a garage independent? Show work
$?P(pool \cap garage) = P(pool)P(garage)? \Rightarrow 0.17? =?(0.64)(0.21) \Rightarrow 0.17 \neq 0.1344 \therefore$ having a pool and a garage are not independent (or they are dependent)

	$P(pool)$	$P(pool')$	
$P(garage)$	0.17	0.47	0.64
$P(garage')$	0.04	0.32	0.36
	0.21	0.79	1

(2) Just after birth, each newborn child is rated on a scale called the Apgar scale. The possible ratings are $0, 1, \ldots, 10$, with the child's rating determined by color, muscle tone, respiratory effort, heartbeat, and reflex irritability (best possible score is 10). Let X be the Apgar score of a randomly selected child born at a certain hospital during the next year. Find the following:

(a) What is the probability that a randomly selected newborn has a score of at most a 7
$P(X \leq 7) = P(0) + P(1) + \cdots + P(7) = 1 - P(X > 7) = 1 - P(X \geq 8) = 1 - [P(8) + P(9) + P(10)]$
$= 1 - (0.25 + 0.12 + 0.01) = 1 - 0.38 = 0.62$

(b) What is the probability that a randomly selected newborn has a score of at least a 4
$P(X \geq 4) = P(4) + P(5) + \cdots + P(10) = 1 - P(X < 4) = 1 - P(X \leq 3) = 1 - [P(0) + P(1) + P(2) + P(3)]$
$= 1 - (0.002 + 0.001 + 0.002 + 0.005) = 1 - 0.01 = 0.99$

(c) Calculate EX, VX, and SDX
$EX = \sum xp(x) = 0(0.002) + 1(0.001) + 2(0.002) + 3(0.005) + 4(0.02) + 5(0.04) + 6(0.18) + 7(0.37) + 8(0.25) + 9(0.12) + 10(0.01) = 7.15$
$VX = \sum (x - EX)^2 p(x) = (0 - 7.15)^2(0.002) + (1 - 7.15)^2(0.001)$
$+(2 - 7.15)^2(0.002) + (3 - 7.15)^2(0.005) + (4 - 7.15)^2(0.02) + (5 - 7.15)^2(0.04)$
$+(6 - 7.15)^2(0.18) + (7 - 7.15)^2(0.37) + (8 - 7.15)^2(0.25) + (9 - 7.15)^2(0.12)$
$+(10 - 7.15)^2(0.01) = 1.5815$
OR $VX = E(X^2) - [E(X)]^2$ with $E(X^2) = \sum x^2 p(x)$
$E(X^2) = 0^2(0.002) + 1^2(0.001) + 2^2(0.002) + 3^2(0.005) + 4^2(0.02) + 5^2(0.04) + 6^2(0.18) + 7^2(0.37) + 8^2(0.25) + 9^2(0.12) + 10^2(0.01) = 52.704 \therefore VX = 52.704 - (7.15)^2 = 1.5815$

$SDX = \sqrt{VX} = \sqrt{1.5815} = 1.257577$

X	0	1	2	3	4	5	6	7	8	9	10
$P(x)$	0.002	0.001	0.002	0.005	0.02	0.04	0.18	0.37	0.25	0.12	0.01

(3) Dr. Peter Venkman wanted to do a test on ESP. He randomly selected his volunteers and they were shown one card of 4 different ones, one card at a time (blank side facing the subject) and were told to guess what shape they thought was on the back side of the card. The test was done for a total of 10 cards per subject.

(a) What is the name of the probability distribution for this? What are the parameters of this distribution?
This is the binomial distribution with parameters $n = 10$ and $p = 0.25$.
The probability is 0.25 because by random chance, you have a 1 in 4 chance of guessing correctly

(b) What is the probability that a random subject can get exactly one card correct?
$P(X = 1) = \binom{10}{1}(0.25^1)(0.75^{10-1}) = 0.1877117$

(c) What is the probability that they will get at least 6 cards correct?
$P(X \geq 6) = P(X = 6) + P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10) = \binom{10}{6}(0.25^6)(0.75^{10-6}) + \binom{10}{7}(0.25^7)(0.75^{10-7}) + \cdots + \binom{10}{10}(0.25^10)(0.75^{10-10}) = 0.0197277$

(d) Suppose that a subject actually has some ESP. Change the probability of success p to 0.5. Now calculate the probability of getting exactly 6 cards correct.
$P(X = 6) = \binom{10}{6}(0.5^6)(0.5^{10-6}) = 0.016222$

(e) Calculate EX, VX and SDX, using both probabilities (so 2 sets of EX, VX, and SDX).
$EX = np = 10(.25) = 2.5$ $VX = npq = 10(.25)(.75) = 1.875$ $SDX = \sqrt{npq} = 1.3693064$
$EX = np = 10(.5) = 5$ $VX = npq = 10(.5)(.5) = 2.5$ $SDX = \sqrt{npq} = 1.5811388$

(4) The number of calls coming per minute into a hotel's reservation center is Poisson random variable with mean 3.

(a) Find the probability that no calls come in a given 1 minute period.
$P(X = 0) = \frac{e^{-3}3^0}{0!} = 0.0497871$

(b) What is the probability that at least 1 call comes in a given 1 minute period?
$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0) = 1 - \frac{e^{-3}3^0}{0!} = 0.9502129$

(c) Calculate EX, VX and SDX
$EX = \mu = 3$, $VX = \mu = 3$, $SDX = \sqrt{\mu} = 1.7320508$

(5) A Quidditch player from the Holyhead Harpies is looking to upgrade her broom. She would like to upgrade from her Nimbus 2001 to the latest Firebolt. She will owl around to various suppliers until she finds one that carries the newest Firebolt. She estimates that the probability of an independent supplier to have the newest Firebolt will be 28%. We are interested in the number of suppliers she will have to owl before she finds one that had the Firebolt she wants

(a) What is the name of this distribution and what is/are the parameter(s)?
This is geometric (owling various suppliers until she finds one that has her broom)
$X \sim geo(p) \Rightarrow X \sim geo(0.28)$ (parameter is $p = 0.28$)

(b) What is the probability that she must owl two suppliers?
$P(X = 2) = q^{x-1}p = (1 - 0.28)^{2-1}(0.28) = 0.2016$

(c) What is the probability that she must owl *at most* four suppliers? $P(X \leq 4) = P(1) + P(2) + P(3) + P(4)$. $P(X = 1) = (0.72)^{1-1}(0.28) = 0.28$,
$P(X = 2) = (0.72)^{2-1}(0.28) = 0.2016$, $P(X = 3) = (0.72)^{3-1}(0.28) = 0.145152$,
$P(X = 4) = (0.72)^{4-1}(0.28) = 0.1045094$ and now $P(X \leq 4) = 0.7312614$

(d) What is the probability that she must owl *at least* two suppliers?
$P(X \geq 2) = 1 - P(X < 2) = 1 - P(X \leq 1) = 1 - 0.28 = 0.72$

(e) On average, how many suppliers would we expect her to have to owl until she finds one that has the newest Firebolt?
$EX = \frac{1}{p} = \frac{1}{0.28} = 3.5714286$

(f) What is the variance and standard deviation?
$VX = \frac{q}{p^2} = \frac{0.72}{(0.28)^2} = 9.1836735$,

$SDX = \sqrt{VX} = \sqrt{9.183673} = 3.0304576$

(6) Each of 12 refrigerators of a certain type has been returned to a distributor because of an audible, high-pitched, oscillating noise when the refrigerators are running. Suppose that 7 of these refrigerators have a defective compressor and the other 5 have less serious problems. If the refrigerators are examined in random order, let X be the number in the random sample (done without replacement) of 6 examined. Of interest are those that have a defective compressor.

(a) What is the name of this distribution and what is/are the parameter(s)?
This is hypergeometric because we have sampling done without replacement:
$N = 12$, $M = 7$, and $n = 6$ so $X \sim hyper(M = 7, N = 12, n = 6)$

(b) What is the probability there will be exactly 4 with a defective compressor?
$P(X = 4) = \frac{\binom{7}{4}\binom{12-7}{6-4}}{\binom{12}{6}} = 0.3787879$

(c) What is the probability there will be at most 4 with a defective compressor?
$P(X \leq 4) = P(4) + P(3) + P(2) + P(1) + P(0)$ where
$P(X = 0) = \frac{\binom{7}{0}\binom{12-7}{6-0}}{\binom{12}{6}} = 0$
$P(X = 1) = \frac{\binom{7}{1}\binom{12-7}{6-1}}{\binom{12}{6}} = 0.0075758$
$P(X = 2) = \frac{\binom{7}{2}\binom{12-7}{6-2}}{\binom{12}{6}} = 0.1136364$
$P(X = 3) = \frac{\binom{7}{3}\binom{12-7}{6-3}}{\binom{12}{6}} = 0.3787879$
$P(X = 4) = \frac{\binom{7}{4}\binom{12-7}{6-4}}{\binom{12}{6}} = 0.3787879$
Now $P(X \leq 4) = 0.8787879$

(d) On average, how many refrigerators would we expect to have a defective compressor?
$EX = \frac{nM}{N} = \frac{(6)(7)}{12} = 3.5$

(e) What is the variance and standard deviation?
$VX = \left(\frac{N-n}{N-1}\right)\left(\frac{Mn}{N}\right)\left(1 - \frac{M}{N}\right)$
$= \left(\frac{12-6}{12-1}\right)\left(\frac{7(6)}{12}\right)\left(1 - \frac{7}{12}\right)$
$= 0.7954545$
$SDX = \sqrt{VX} = \sqrt{0.7954545} = 0.8918826$

(7) Conditional probabilities can be useful in diagnosing disease. Suppose that three different, closely related diseases ($A_1,\ A_2,\ and\ A_3$) occur in 25%, 15%, and 12% of the population. In addition, suppose that any one of the three mutually exclusive symptom states ($B_1,\ B_2, and\ B_3$) may be associated with each of these diseases. Experience shows that the likelihood $P(B_j|A_i)$ of having a given symptom state when the disease is present is as shown in the following table. Find the probability of disease A_2 given symptoms $B_1,\ B_2,\ B_3,\ and\ B_4$, respectively.

	Disease State		
Symptom state	A_1	A_2	A_3
B_1	0.08	0.17	0.10
B_2	0.18	0.12	0.14
B_3	0.06	0.07	0.08
B_4	0.68	0.64	0.68

Use Bayes' theorem:

$$P(A_i|B_j) = \frac{P(B_j|A_i)P(A_i)}{\sum P(B_j|A_i)P(A_i)}$$

Use the Law of Total Probability to find the denominator, B_j; there will be 4 different ones because there are 4 states of symptoms.

$P(B_1) = P(B_1|A_1)P(A_1)+(B_1|A_2)P(A_2)+(B_1|A_3)P(A_3) = (0.08)(0.25)+(0.17)(0.15)+(0.10)(0.12) = 0.0575$
$P(B_2) = P(B_2|A_1)P(A_1)+(B_2|A_2)P(A_2)+(B_2|A_3)P(A_3) = (0.18)(0.25)+(0.12)(0.15)+(0.14)(0.12) = 0.0798$
$P(B_3) = P(B_3|A_1)P(A_1)+(B_3|A_2)P(A_2)+(B_3|A_3)P(A_3) = (0.06)(0.25)+(0.07)(0.15)+(0.08)(0.12) = 0.0351$
$P(B_4) = P(B_4|A_1)P(A_1)+(B_4|A_2)P(A_2)+(B_4|A_3)P(A_3) = (0.68)(0.25)+(0.64)(0.15)+(0.68)(0.12) = 0.3476$

$$P(A_2|B_1) = \frac{(0.17)(0.15)}{0.0575} = 0.4434783$$

$$P(A_2|B_2) = \frac{(0.12)(0.15)}{0.0798} = 0.2255639$$

$$P(A_2|B_3) = \frac{(0.07)(0.15)}{0.0351} = 0.2991453$$

$$P(A_2|B_4) = \frac{(0.64)(0.15)}{0.3476} = 0.2761795$$

(8) You randomly draw cards from a deck until you get four aces (with replacement).

(a) What is the name of this distribution and what is/are the parameter(s)?
Negative binomial since you are wanting to know how many times you have to draw to get 4 aces.
$X \sim nb(r = 4, p = \frac{1}{13})$

(b) What is the chance that you will draw exactly 20 times to find 4 aces?
$P(X = 20) = \binom{20-1}{4-1}(\frac{1}{13})^4(\frac{12}{13})^16 = 0.0094266$

(c) Calculate EX, VX, and SDX
$EX = \frac{r}{p} = \frac{4}{1/13} = 52$, $VX = \frac{rq}{p^2} = \frac{4(12/13)}{(1/13)^2} = 624$, $SDX = \sqrt{VX} = \sqrt{624} = 24.979992$